AMSD Remote Interferometric Radius of Curvature Measurement

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AMSD Requirements

- AMSD parent paraboloid has 10 m vertex RoC.
- AMSD segment centered 1400 mm off-axis with flat-to-flat diameter of 1200 mm.
- SOW requires parent vertex RoC = 10 m ± 1 mm.
- MSFC needs capability to verify compliance of AMSDs at cryo temperatures.
- Need capability to measure vertex RoC for a segment with no physical vertex using no vertex fixtures with accuracy of about one part in 10^5.

Shape Measurement of 2-D Surfaces

- A surface can be defined as a function of two coordinates x_1 and x_2 and of several shape parameters c_1, c_2, …, c_n
- A conic of rotation, in Cartesian pupil coordinates, could be a function of coordinates x and y as well as a function of the radius of curvature (R) and conic constant (K).
- Shape measurement consists of establishing the values of the shape parameters which describe the surface that "best" matches the test surface in a given coordinate system.

Shape Measurement Step 1: Acquisition of Surface Profile

- A representation of the surface is generated using some kind of measurement device (a coordinate measuring machine, profilometer, interferometer, etc.).
- The result is a discrete representation consisting of a set of ordered triplets $\{(x_i, y_i, z_i)\}$ that describe the locations of points on the surface in the chosen coordinate system.

Shape Measurement Step 2: Regression of Shape Parameters

- A function is chosen *a priori*, up to a set of free shape parameters $\{c_i\}$, to represent the surface mathematically.
- The choice of model function depends on how the experimenters wish the define the test surface.
- A fitting criterion, $D[\{f(x_i, y_i; c_1, c_2, \ldots, c_n), z_i\}]$, is chosen to give meaning to "best" fit.

Shape Measurement Step 2: Regression of Shape Parameters

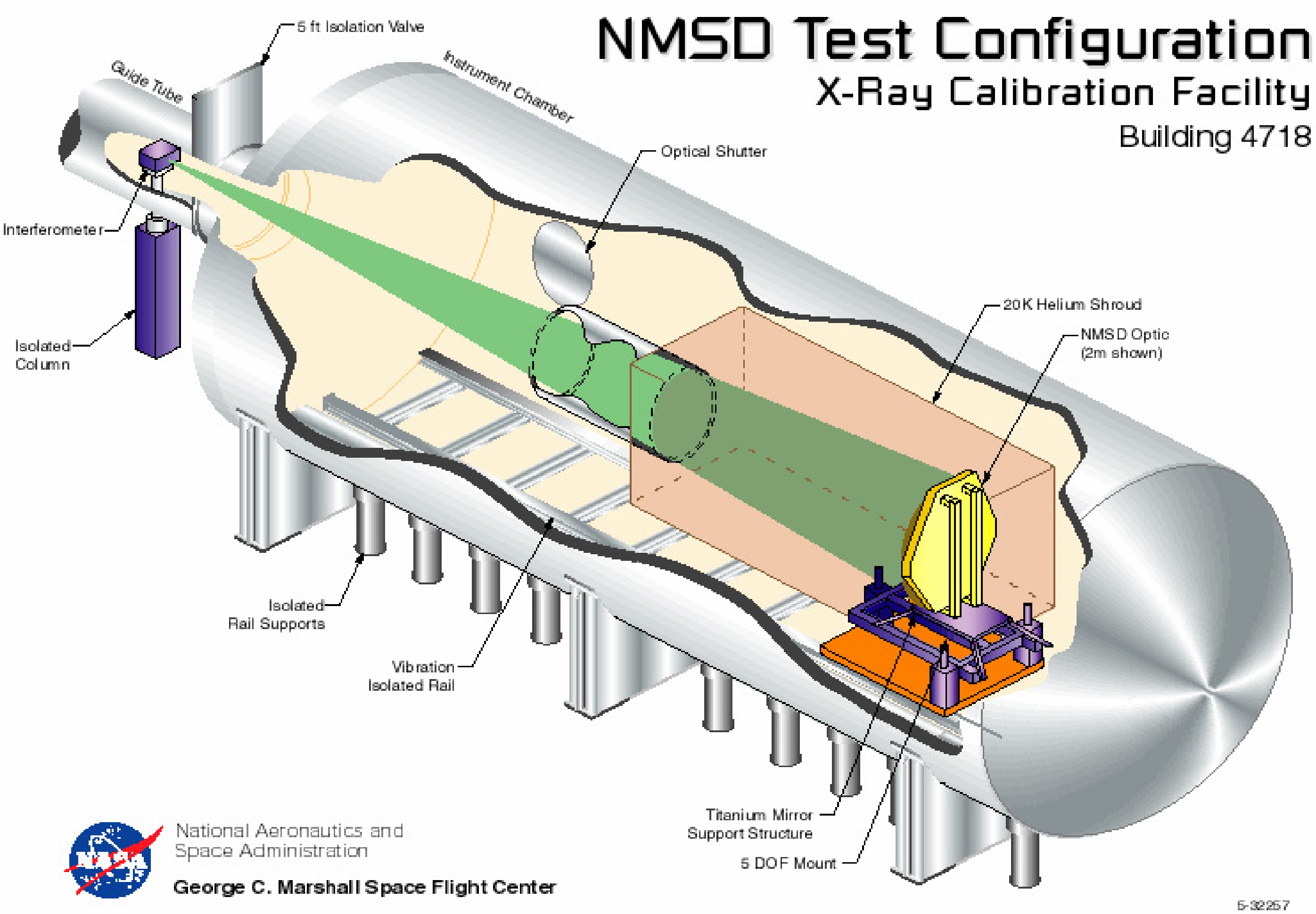
- If a least-squares fit is desired, for example, then the criterion becomes

$$D = \min\left[\sum_i \left(f_i - z_i\right)^2\right]$$

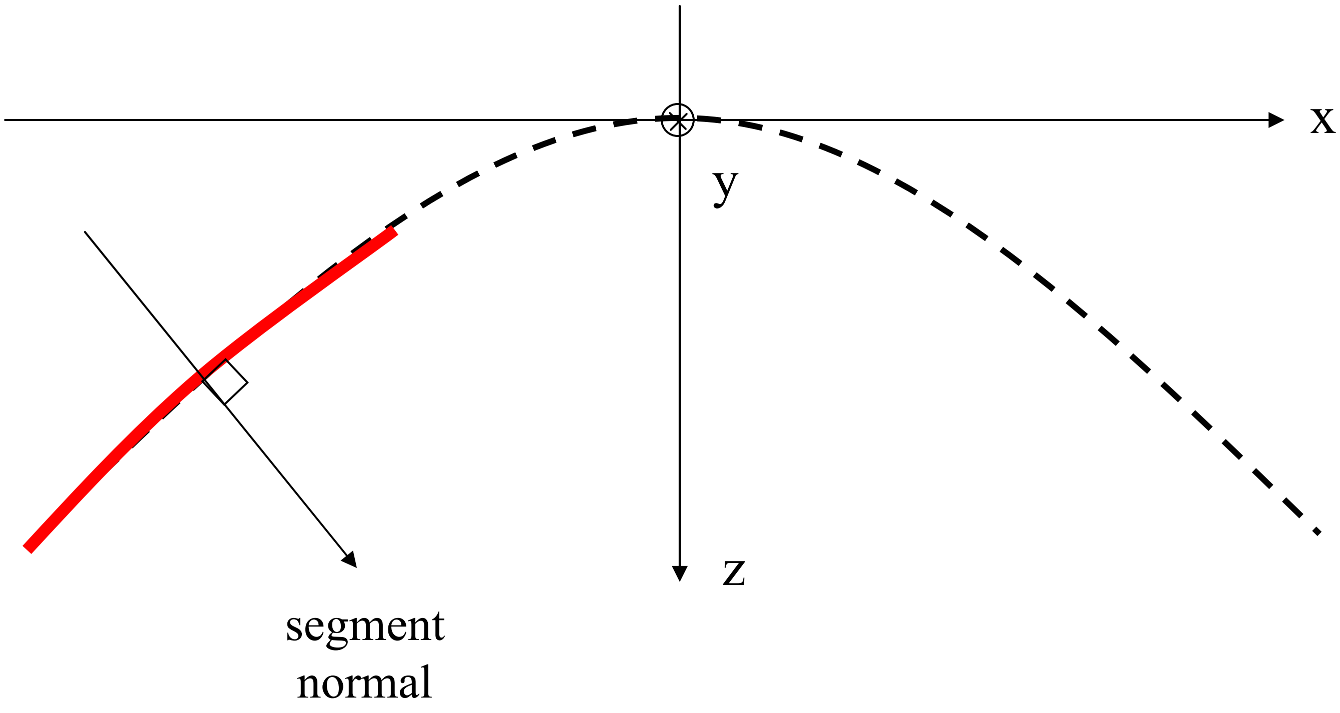
- Two things, in general, are varied in order to fulfill this criterion: 1) the shape parameters $\{c_i\}$ and 2) the rigid body orientation of the surface.

- The degrees of freedom in the orientation of the surface are also a matter of definition and must be decided upon by the experimenters *a priori.*

AMSD Test Configuration



AMSD Parent Coordinate System



RoC Measurement Method

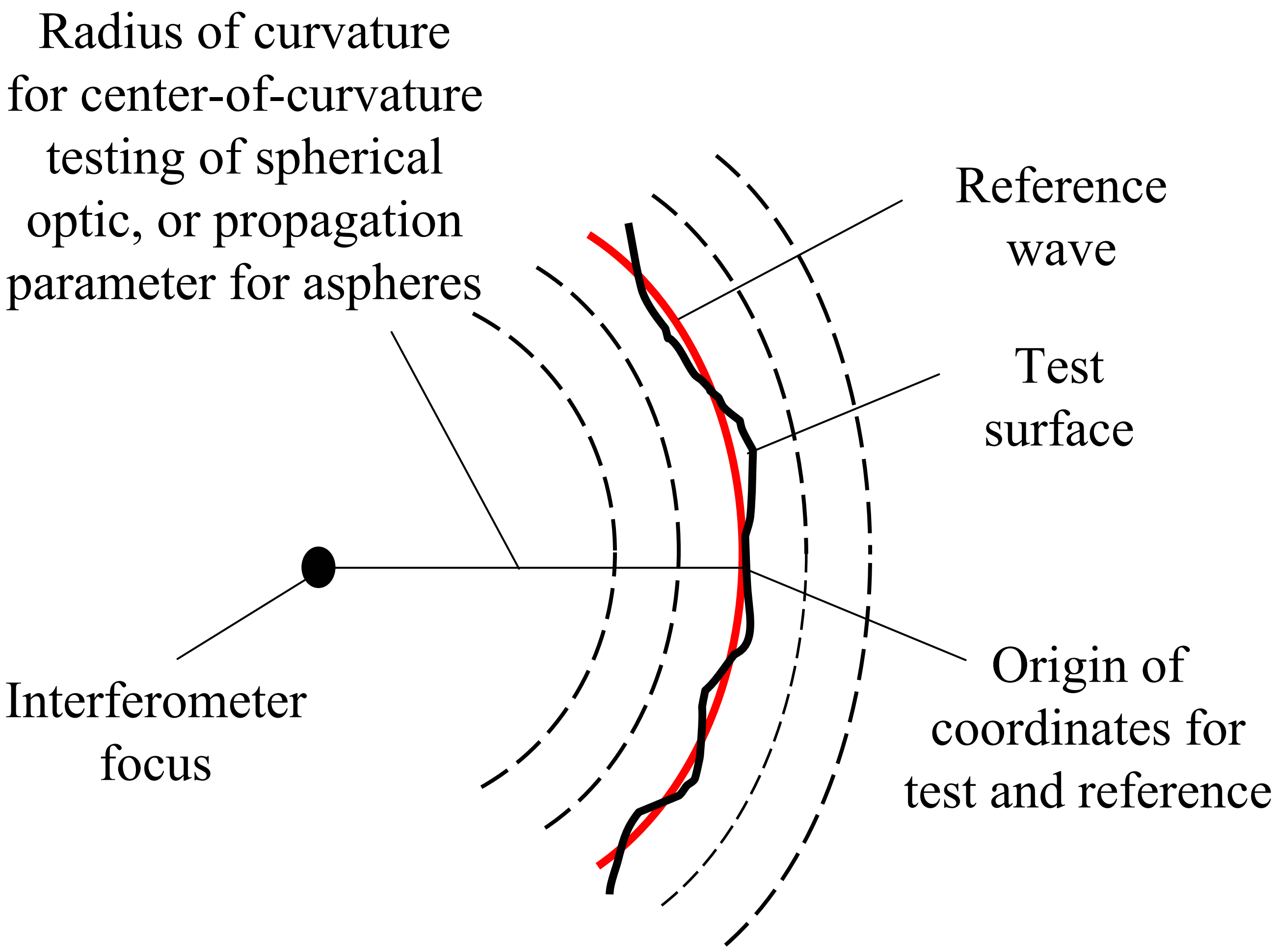
The three basic steps in measuring AMSD vertex RoC:

1. An OPD measurement is performed using an instantaneous phase-shifting interferometer in a center-of-curvature null coniguration. The mirror does not need to be optimally aligned.

2. The OPD data is transformed into a surface profile in parent coordinates. This is done by first changing the OPD into a difference in sagitta between the test surface and the reference wave. The difference is then added to the reference itself to yield a discrete set of data representing the absolute surface of the AMSD test mirror.

3. A paraboloid is fit to the absolute surface data with the vertex RoC as the optimization variable using a least-squares approach.

OPD and Defocus Measurement



Locating the OPD Data in Lab Coordinates

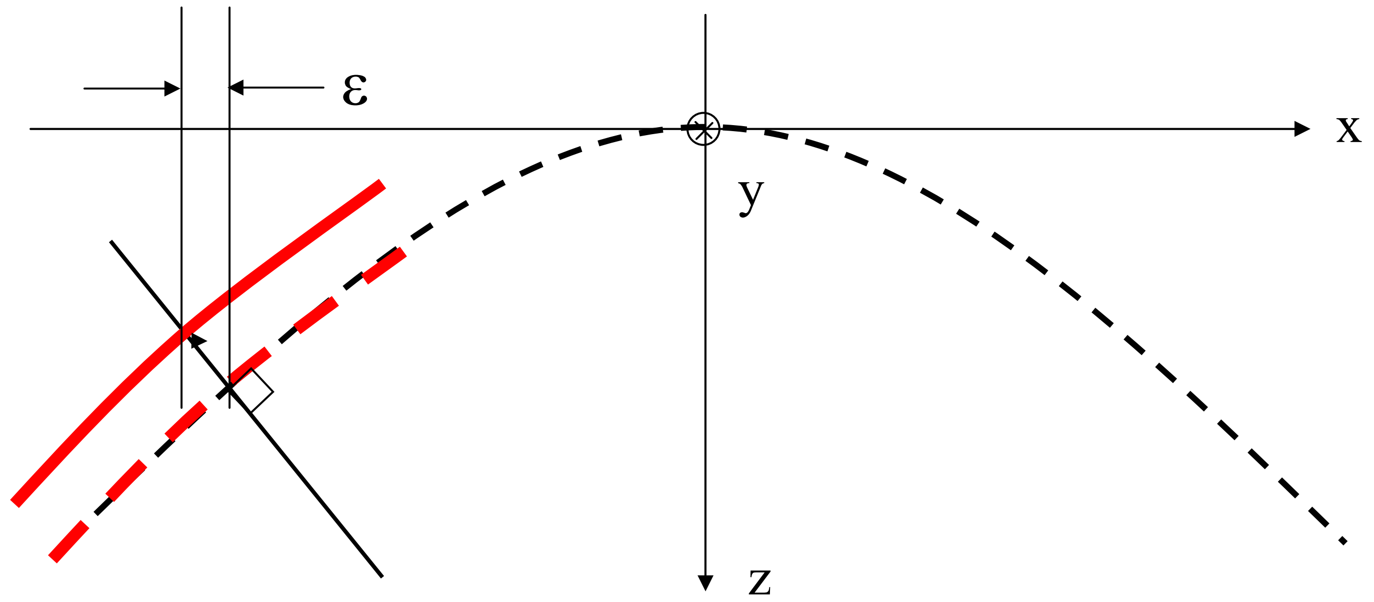
The locations $\{(x_i,y_i)\}$ of the OPD data in lab coordinates must be established before the surface profile can be represented.

1. Each pixel in the interferometer output corresponds to a physical location on the null optic. The pixel indices of the data are transformed into physical null coordinates.

2. The image of the fringe system is distorted by the null in a known way. A transformation is applied which carries the OPD, as it appears mapped onto the null, into the lab coordinates.

Locating the OPD Data in Lab Coordinates

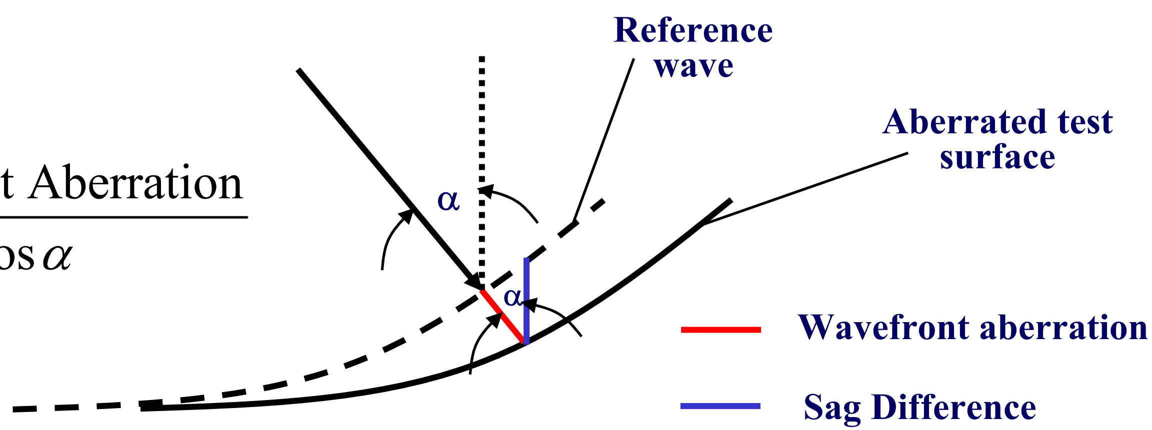
3. Defocus of the off-axis segment (along its central normal) causes a small lateral shift of the segment in parent coordinates. This must be accounted for in establishing the pupil coordinates $\{x_i, y_i\}$ of the OPD data.



Transformation of Wavefront Aberration into Sag Difference

Wavefront aberration is transformed into sag difference on the assumption that the test surface normals are, to first order, the same as the reference surface normals and that wavefront aberration is a slowly varying function of pupil coordinate.

$$\text{Sag Difference} = \frac{\text{Wavefront Aberration}}{\cos\alpha}$$



Generating the Surface Profile

After transforming the wavefront aberration into a difference in sagitta, in parent coordinates, between the test mirror and the reference wave, a discrete set of data representing the absolute AMSD surface is obtained simply by adding this difference to the reference wave itself. A surface of any desired form can then be fit to this data.

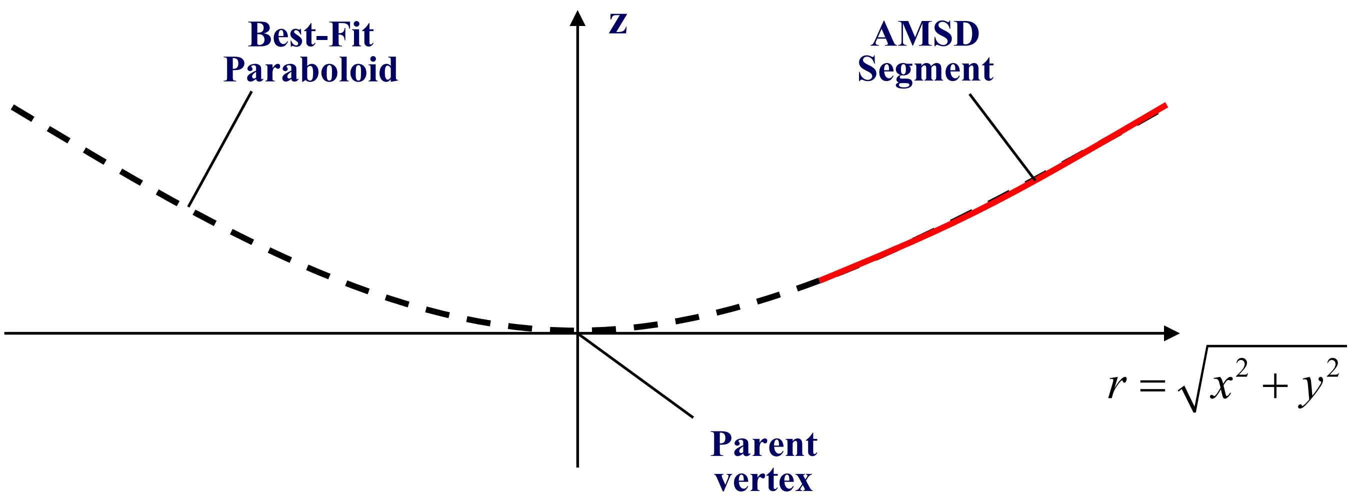
$$z_i = (\text{sag difference})_i + (\text{reference wave})_i$$

$$= (\text{mirror}_i - \text{reference wave}_i) + (\text{reference wave})_i$$

Now we have the set of ordered triplets $\{(x_i, y_i, z_i)\}$ that form a discrete representation of the mirror surface.

Curve Fit to Paraboloid

The discrete AMSD surface sag data is now in parent coordinates, with the parent optical axis at coordinates (x,y) = (0,0). A paraboloid is fit to this sag data. A least-squares algorithm directly yields the best-fit RoC.



Radial slice of AMSD segment and best-fit paraboloid

RoC is a Free Parameter

There are two parameters in our model function. One is simply a constant term to allow the sag of the best-fit paraboloid to depart from zero at the optical axis. The other parameter is a function of the RoC only. We minimize the sum-square error between the data and the fitting function (a paraboloid) by optimizing the RoC.

$$E = \sum_{i=1}^{N} \left[f\left(r_i\right) - z_i \right]^2$$

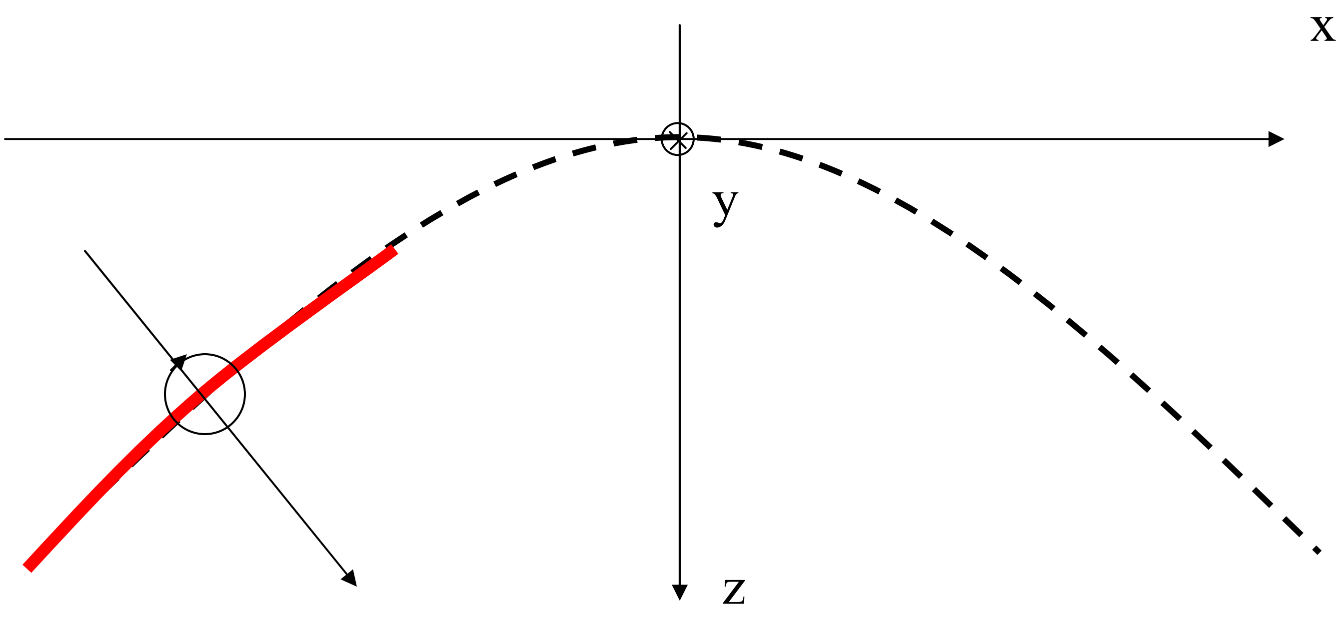
The sum-square error (the quantity to be minimized). r_i is radial coordinate, z_i is the sag at coordinate r_i, and N is the total number of sag data points.

$$f\left(r_i\right) = a_0 + \frac{r_i^2}{2\mathrm{g}RoC}$$

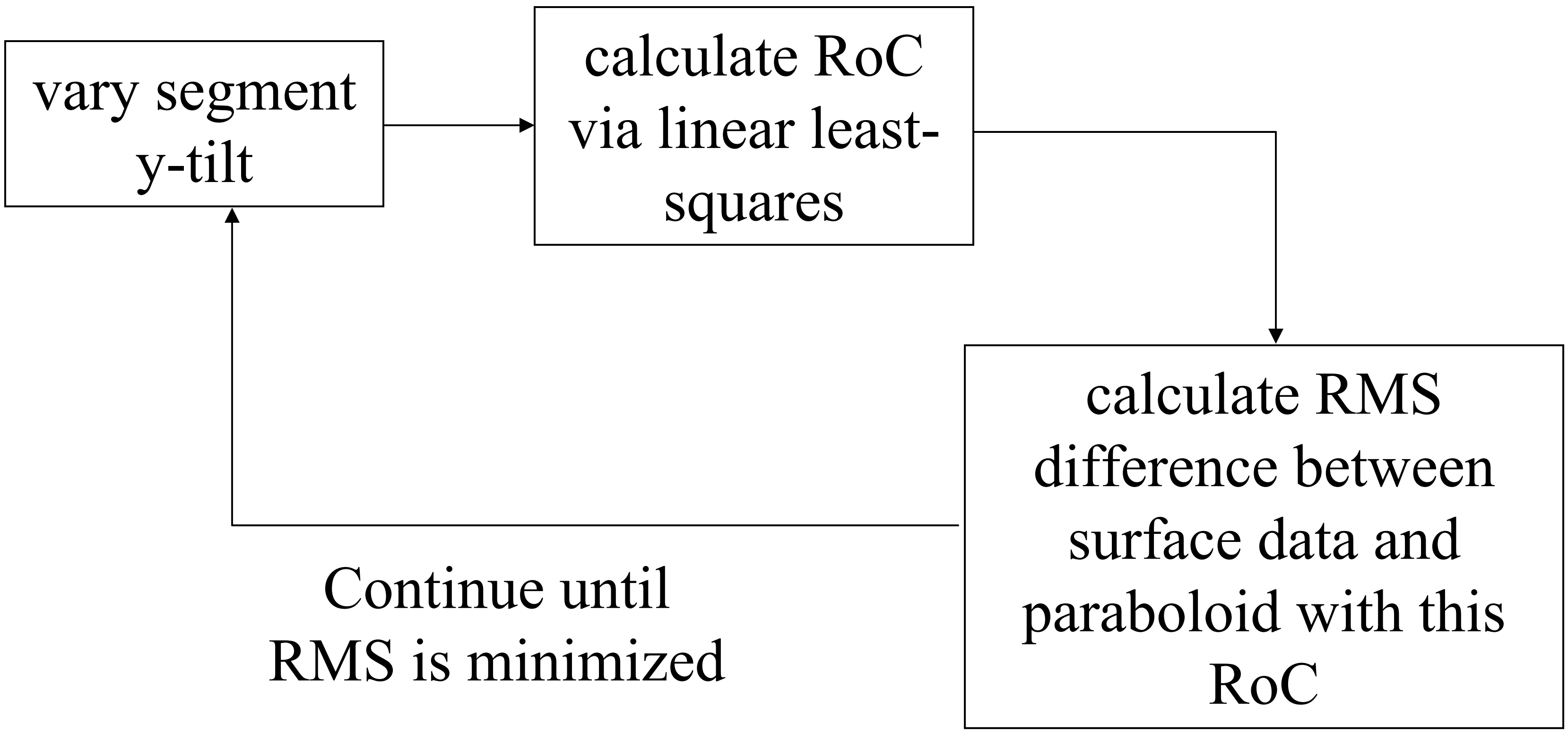
The explicit form of the function to which we are fitting. It is a paraboloid.

Varying the X-tilt Orientation of the Mirror

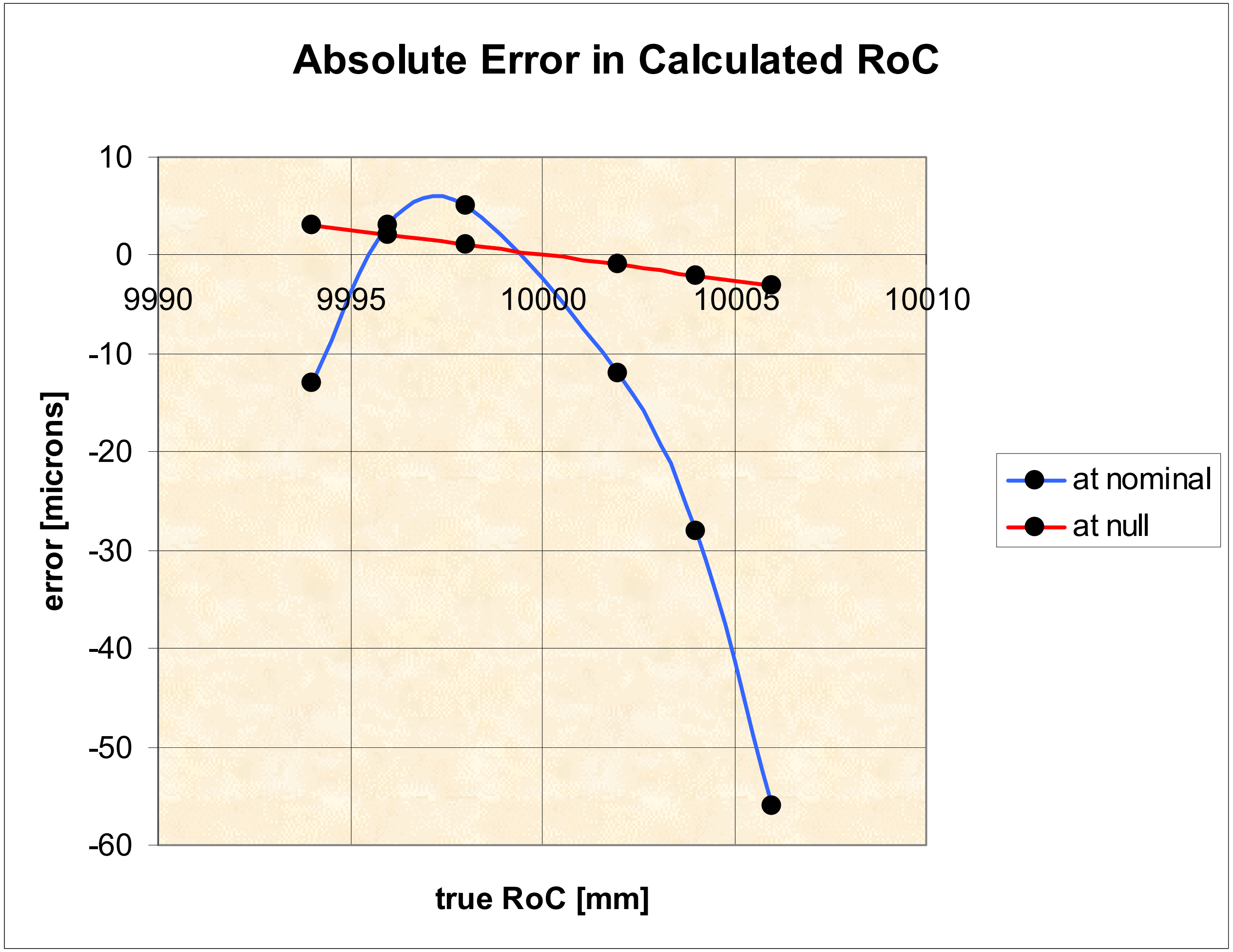
In addition to varying the base radius of the model paraboloid, we let the mirror rotate about an axis, parallel to the parent y-axis, which passes through the center of the segment.



The Fitting Algorithm



Simulations Using Code V Generated Data



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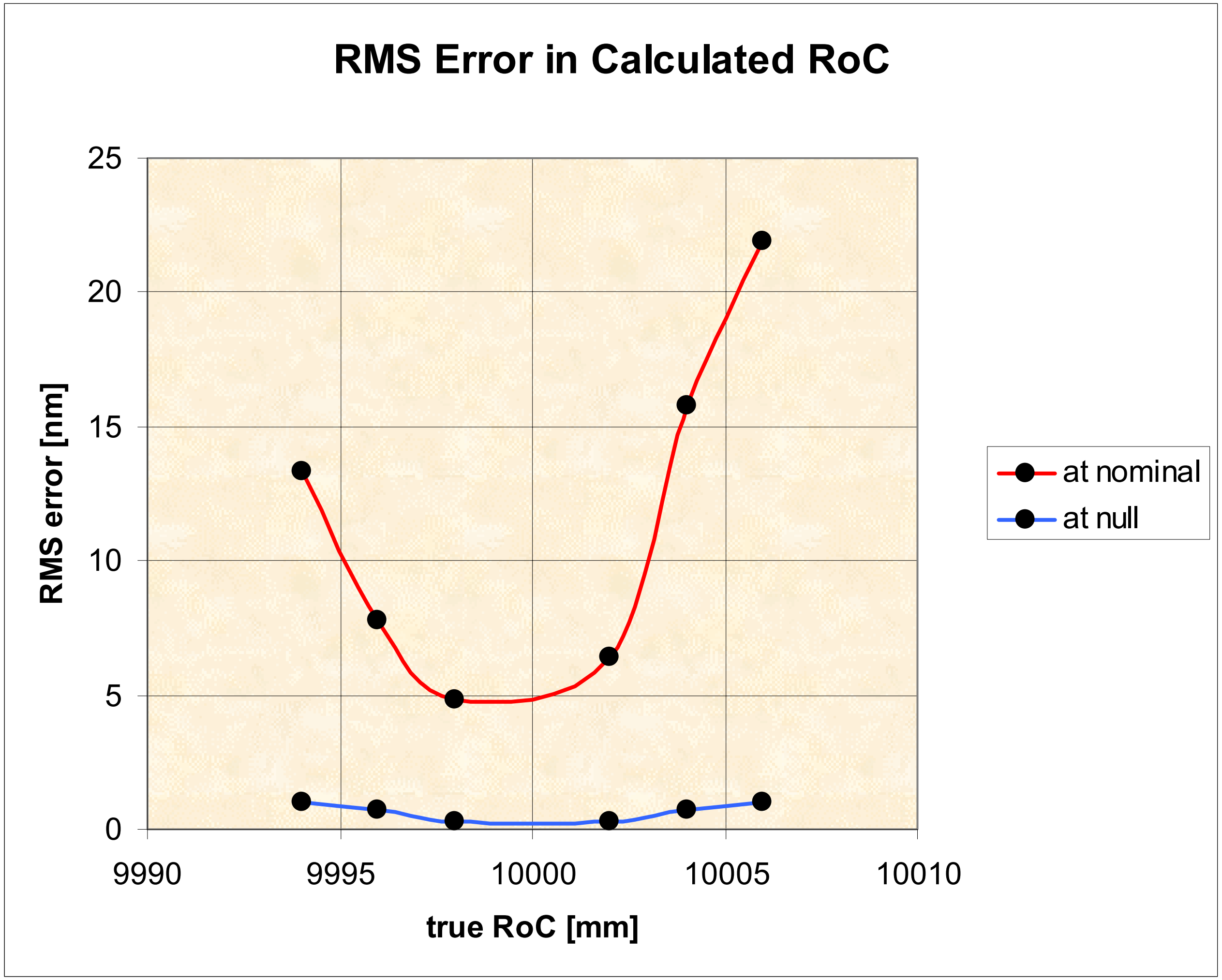
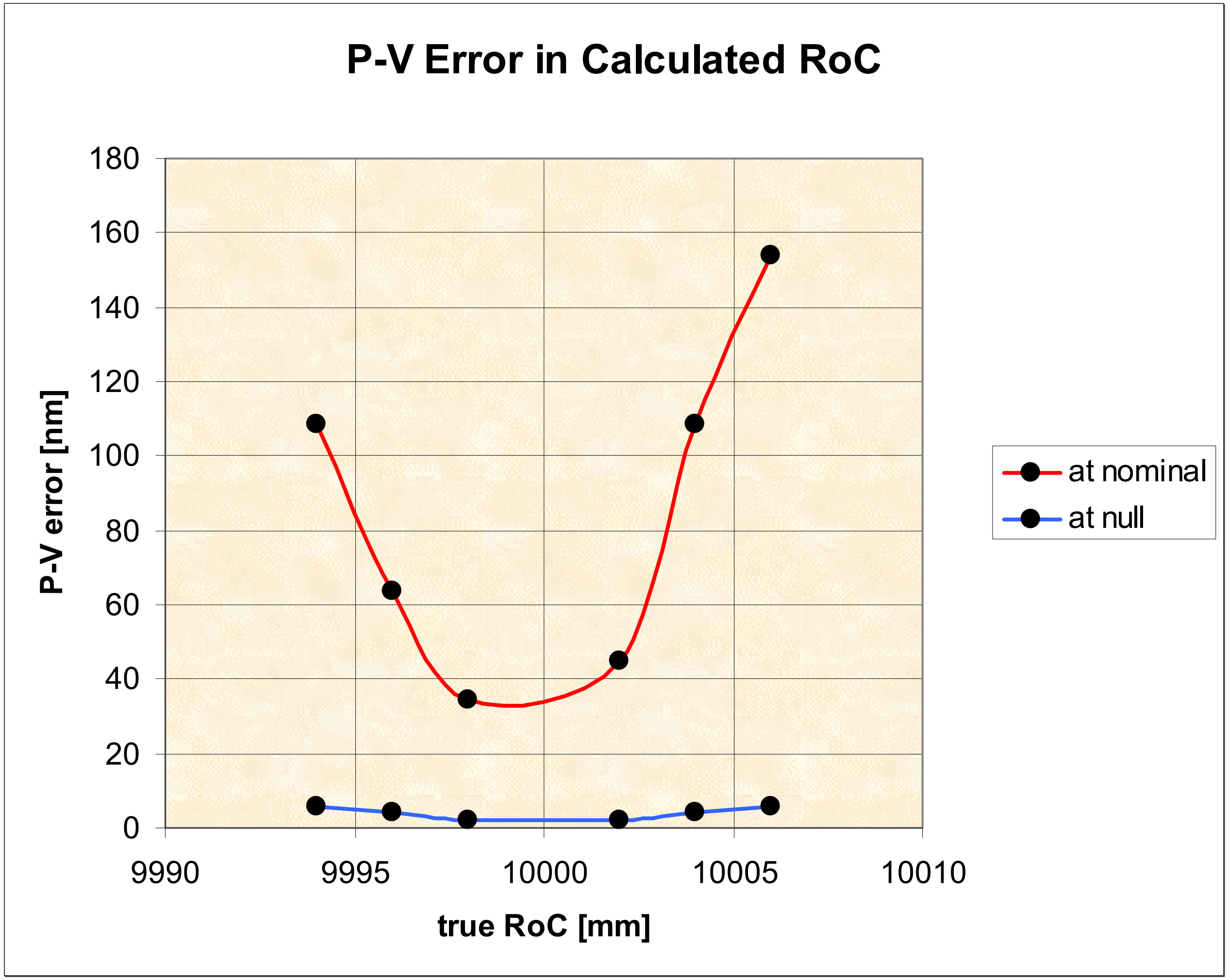
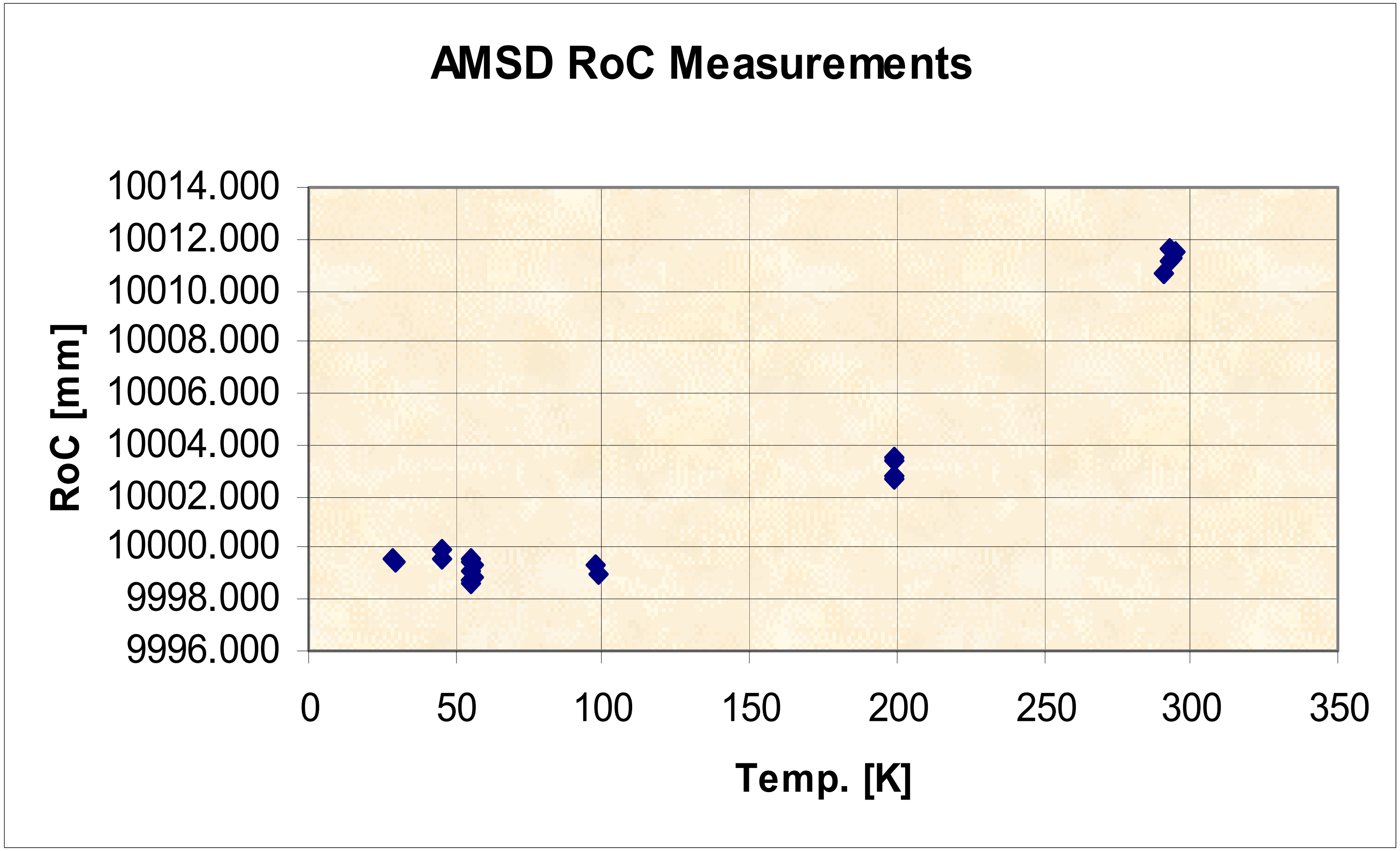


Simulations Using Code V Generated Data



AMSD RoC Measurements



Conclusions

A remote method for measuring radius of curvature has been devised which requires no more instrumentation than that used to measure mirror figure interferometrically. The method has been shown to work extraordinarily well using simulated data that contains errors that are meant to simulate errors encountered in live testing. The fitting algorithm was tailored to provide good answers for these simulated data sets. More work, however, needs to be done to make the method more accurate for the live tests, where the coordinates of the data may change from test to test, though the mirror shape itself may remain the same.